CAUCHY'S PROBLEM AND THE PROBLEM OF A PISTON FOR ONE-DIMENSIONAL, NON-STEADY MOTIONS OF GAS (AUTOMODEL MOTION)*

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Considered here is the automodel problem for the equations of one-dimensional, non-steady motions of ideal non-heat-conducting perfect gas (Cauchy's problem) and the problem of automodel motions of gas with properties mentioned above, created by symmetrical expanding piston contained in the gas.

Automodel presentation of the first problem was first given by Sedov [1]. Special cases of this problem were considered by Sedov [2] (problem concerning the focusing of gas in one point and about its expansion from a point) and Kompaneetz [4].

The second problem, for the case where the velocity of the piston is constant, was discussed by Sedov [2]. The case when the piston velocity is a power function of time was dealt with by Krasheninnikova [3].

In this article we show that: (1) For certain given initial functions, the solution of Cauchy's problem cannot be continued for all instants of time t. (2) Under certain values of the exponent in the formula for piston velocity, the solution for the second problem does not exist.

* Translator's Note. It is known that the problem on one-dimensional motion of gases can be solved by integrating related partial differential equations. If, among parameters which define problem of motion, besides linear coordinates r and time t there are only two parameters (constants) with independent dimensions, then these partial differential equations can be reduced to ordinary differential equations. The corresponding motion of gases bears the name of "automodel motion". [1] p. 149, [3], p. 22.

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1. Cauchy's problem. We have the following equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial r}=0,\quad \frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial r}+\frac{(\nu-1)\rho u}{r}=0$$
$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^{\gamma}}\right)+u\frac{\partial}{\partial r}\left(\frac{P}{\rho^{\gamma}}\right)=0 \tag{1.1}$$

The initial conditions are:

$$P(r,0)=L_1Ar^{\alpha},\quad \rho(r,0)=N_1Br^{\beta},\quad u(r,0)=M_1\sqrt{\frac{A}{B}}\,r^{\frac{\alpha-\beta}{2}}\qquad (r>0) \tag{1.2}$$

For motions with plane waves likewise

$$P(r,0)=L_2A(-r)^{\alpha},\qquad \rho(r,0)=N_2B(-r)^{\beta}$$
$$u(r,0)=M_2\sqrt{\frac{A}{B}}\,(-r)^{\frac{\alpha-\beta}{2}}\qquad (r<0) \tag{1.3}$$

Where α, β, L_i, N_i, $M_i (i = 1,2)$ are dimensionless constants and A and B dimensional constants.

Problem (1.1), (1.2), (1.3) is automodel [1,2] and its solution is reduced to integrating a system of ordinary differential equations

$$\frac{dz}{dV}=\frac{z}{(V-q)W}\{[2(V-1)+\nu(\gamma-1)V](V-q)^2-$$
$$-(\gamma-1)V(V-1)(V-q)-[2(V-1)+\varkappa(\gamma-1)]z\} \tag{1.4}$$

$$q\frac{d\ln\lambda}{dV}=\frac{(V-q)^2-z}{W} \tag{1.5}$$

$$(V-q)\frac{d\ln R}{d\ln\lambda}=q\frac{W}{z-(V-q)^2}+q(\beta+\nu)V \tag{1.6}$$

where

$$W=V(V-1)(V-q)+(\varkappa-\nu V)z,\quad q=\frac{2}{2-(\alpha-\beta)},\quad \varkappa=-\frac{\alpha}{\gamma}q, \tag{1.7}$$

$$u(r_1t)=\frac{r}{t}V(\lambda),\qquad p(r_1t)=B|r|^{\beta+2}t^{-2}P(\lambda)$$
$$\rho(r_1t)=B|r|^{\beta}R(\lambda),\qquad z=\gamma\frac{P}{R},\qquad \lambda=\sqrt{\frac{A}{B}}\,t\,|r|^{\frac{\alpha-\beta}{2}-1} \tag{1.8}$$

Cauchy's problem, i.e. problem (1.1), (1.2) or (1.1), (1.2) and (1.3) has only one analytical solution near any finite point of the $t = 0$ axis, except possibly point $r = 0$, according to theorem of Kovalevskaia. Therefore, close to $\lambda = 0$, a unique solution analytical in λ exists for the system (1.4), (1.5) and (1.6).

At $t \to 0$ from (1.8) we arrive at expressions of the type (1.2), i.e. functions V, P, R at $\lambda \to 0$ will have the following expansions:

$$V=M_1\lambda+V_2\lambda^2+\cdots,\quad P=L_1\lambda^2+P_3\lambda^3+\cdots,\quad R=N_1+R_1\lambda+\cdots \tag{1.9}$$

It follows that the integral curve of equation (1.4) which corresponds to the solution of Cauchy's problem, will pass through the point $V = z = 0$ where it will have an asymptotic representation

$$z = \frac{\gamma L_2}{N_1 M_1^2} V^2 + \dots \quad \text{for } M_1 \neq 0, \quad N_1 \neq 0$$
$$z = -\frac{\gamma}{\alpha} V + \dots \quad \text{for } M_1 = 0, \text{ or } N_1 = 0 \qquad (1.10)$$

in addition for motions with plane waves we will have:

$$V = -M_2\lambda + \dots, \qquad P = L_2\lambda^2 + \dots, \qquad R = N_2 + \dots \qquad (1.11)$$

$$z = \frac{\gamma L_2}{N_2 M_2^2} V^2 + \dots, \quad \text{or} \quad z = -\frac{\gamma}{\alpha} V + \dots. \qquad (1.12)$$

In this manner, two integral curves correspond to the solution of Cauchy's problem. These curves, close to the point $V = z = 0$ have asymptotic representations (1.10) and (1.12).

2. To aid in the solution of our problem it is necessary to conduct a preliminary qualitative study of the field of integral curves of equation (1.4). From this study we shall establish the path for an integral curve, originating from formulas (1.10) or (1.12) for small values of V and z, and the point of its termination for a complete solution of our problem.

If the integral curve is known, then with the help of equation (1.5) we can calculate the distribution of λ along this curve.

Displacement along the integral curve from point $V = z = 0$ should cause a monotonic increase of parameter λ. This increase continues to infinity or to some finite value, corresponding to the moving boundary of the region, occupied by the gas, if such a boundary arises as in the case of a moving piston.

In this manner, Cauchy's problem will be solved if the corresponding integral curve is so constructed that, moving along the curve parameter λ, it goes monotonically through the set of values indicated above.

In the most general case of automodel motion, with fixed values of γ and ν, the field of integral curves and the character of the variation of λ along them can be determined by only two parameters q and κ [see (1.4) and (1.5)]. In the case of Cauchy's automodel problem, these parameters are expressed through α and β in formulas (1.7) which have unique solutions for α and β

$$\alpha = -\gamma\varkappa / q, \qquad \beta = -\gamma\varkappa / q - 2(1 - 1/q)$$

From here we can see that the field of integral curves and the distribution of λ along them for an arbitrary automodel motion are simultaneously valid for some automodel problem of Cauchy. By using well-studied fields of integral curves of several special automodel problems, it is easy to describe a corresponding Cauchy problem.

In the case of a strong explosion, we have:

$$q = 2/(2+\nu), \quad \varkappa = 2\nu/\gamma(2+\nu),$$

which makes $\alpha = -\nu$, $\beta = 0$.

In the case of problems of attenuation of an arbitrary explosion, in the case of a piston moving with a constant velocity, in the propagation of a flame front and a detonation, in the case of focusing at a point and expansion from a point in a gas [2] we have $q = 1$, $\kappa = 0$ which makes $\alpha = 0$, $\beta = 0$. Cauchy's problem in this case is the first one in the case of motion with plane waves and the last one among the ones mentioned above. This problem is developed completely and in every detail by Kochin and Sedov [2].

In the case of a strong explosion in a medium of variable density [2] we have

$$q = 2/(5-\omega), \quad \varkappa = 6/\gamma(5-\omega),$$

which makes $\alpha = -3$ and $\beta = -\omega$. Here, ω is the exponent in the formula for the initial exponential distribution of density along the coordinate axis. In the case of the problem of the motion of gas [3] compressed by a symmetrical piston, moving with a velocity $U = ct^n$ we shall have $q = n + 1$, $\kappa = -2n/\gamma$ and this gives

$$\alpha = 2n/(n+1), \quad \beta = 0.$$

Finally in Ref. [4], Cauchy's problem is considered for the case of motion with plane waves with $\alpha = a$, $\beta = 0$ and for the following special values of the dimensionless constants in formulas (1.2) and (1.3): $M_1 = M_2 = L_2 = 0$, $N_1 = N_2 = L_1 = 1$.

The investigation of the field of integral curves, without assigning concrete values to parameters q and κ for $\nu \neq 1$ presents quite a difficult problem because for the solution of the problem, or to establish coordinates for some singular points, it is necessary to establish the number of real roots of a certain cubic equation, the coefficients of which depend on q and κ in complicated manner, and to solve for these roots.

However, for $\mu = 1$, the cubic equation degenerates into a quadratic, and the roots then assume a simple form. In this manner, for motions with plane waves, the coordinates of all singular points are known in the form of functions q and κ, and the problem of the integral curve field investigation is greatly simplified.

Characteristic isoclines are:

$$\text{for } dz/dV = 0 \qquad (2.1)$$

$$z = 0, \quad z = (V-q)\frac{(V-q)[2(V-1)+\nu(\gamma-1)V]-(\gamma-1)V(V-1)}{2(V-1)+\varkappa(\gamma-1)} = f(V)$$

for $dz / dv = \infty$

$$V = q, \quad z = \frac{V(V-1)(V-q)}{\nu V - \varkappa} = f_2(V) \tag{2.2}$$

The points of intersection of these isoclines are singular points. In particular the singular points are the points of intersection of curves $z = f_1(V)$ and $z = f_2(V)$ (the coordinates of other singular points are obvious). By dividing the relation $f_1(V) = f_2(V)$ through by $V - q$ we obtain a cubic equation. In the case of $\nu = 1$, this equation will have the following roots

$$V_1 = \frac{2}{\gamma+1}, \qquad V_2 = \frac{q\varkappa}{q+\varkappa-1} \tag{2.3}$$

Corresponding values of z are

$$z_1 = -\frac{2(\gamma-1)[2-(\gamma+1)q]}{(\gamma+1)^2[2-(\gamma+1)\varkappa]}, \qquad z_2 = \left[\frac{q(q-1)}{q+\varkappa-1}\right]^2 \tag{2.4}$$

Note that singular point $V = z = 0$ is a node in the most general case. The fan of integral curves emanating from this point, because of asymptoti relations (1.10) and (1.12), corresponds to any given initial distribution Therefore, to solve Cauchy's problem for all initial distributions, every curve out of this bundle has to be traced through to a point where $\lambda = \infty$ or where $\lambda = \lambda_\pi$ where λ_π is the value of λ arising in the process of the motion of the boundary of the gas. It is known [2], that in the plane V, z points corresponding to such a boundary can only be the points located on the line $V = q$. Thus the problem is to trace the integral curves of the fan to some special points like $V = q$, $z = 0$, $z = \infty$ or points at which $\lambda = \infty$ (including infinitely distant singular points). In addition λ must monotonically change along the integral curves.

It is known [2] that on the plane V, z exists a parabola

$$z = z_1 = (V-q)^2 \tag{2.5}$$

on which, while moving along the integral curves that cross it, the parameter λ reaches a stationary value i.e. $\partial\lambda/\partial s = 0$ (where s is the length of the arc of the integral curve). This means that if on parabola (2.5) λ assumes a maximum or a minimum value λ_0, then as with a fixed r(or t) variations of λ correspond to variations of t(or r), then transitions along the integral curve over parabola (2.5) correspond to folding of integral surfaces $u = u(r,t)$, $p = p(r,t)$, $\rho = \rho(r,t)$ back into the region of parameters covered previous to the transition across the parabola (2.5), that is the solution is discontinuous for all instants of time $t > 0$. The line $\lambda = \lambda_0$ is the "limiting line" determining the region of corresponding continuous solutions. The intersection of the integral curve with the parabola does not always mean that the solution cannot be extended beyond the line $\lambda = \lambda_0$ It is often possible to cross the parabola (2.5) in a jump which corresponds to discontinuous-solution-motions with shock waves. In all the previously discussed cases of automodel motion

[2-4] the discontinuous solution was possible. This solution was set up in a unique manner.

But sometimes it is impossible to set up either the continuous or the discontinuous type of solution; this may take place in the solution of Cauchy's problem as is shown below.

3. Consider Cauchy's problem for which the field of integral curves is the field of problems of a strong explosion.

Case 1, $\nu \neq 1$. Careful study of the field of integral curves for $z > 0$ reveals the following picture (Fig. 1). All of the integral curves, emanating in a bundle from the origin of the coordinate system enter the singular point $V = q = 2/(2 + \nu)$, $z = 0$ where they have a slope $-\gamma/(2 + \nu)$ remaining integral curves going through the point $V = 2/(2 + \nu)$; $z = 0$ except for straight lines $z = 0$ and $V = 2/(2 + \nu)$ have the same slope at this point. All except one of them go into the singular point $V = 2/(2 + \nu)$, $z = \infty$.

The noted exceptional integral curve corresponds to the solution of the problem of a strong discontinuity and at $z \to \infty$ enters into a singular point (saddle) $V = 2/\{\gamma(2 + \nu)\}$, $z = \infty$. The equation of this curve is [2]:

$$z = \frac{(\gamma - 1) V^2 [V - 2/(2 + \nu)]}{2[2/\gamma(2 + \nu) - V]} \tag{3.1}$$

The arrows on the integral curves indicate the direction of increasing λ.

From Fig. 1 we can see that as all integral curves come from the origin of the coordinates, cross parabola (2.5) (on which λ reaches its maximum, but still remain finite); the continuous solution of Cauchy's problem cannot be continued for all values of $t > 0$. Discontinuous solutions of the problem, continuous for all values of $t > 0$ could be set up if a jump transition corresponding to the conditions of the shock wave could be made from the integral curves emanating from the origin to those emanating from point $V = 2/(2 + \nu)$, $z = 0$ and entering point $V = 2/(2 + \nu)$; $z = \infty$. Let us show that this is impossible.

Because the relations on the shock wave map the region between the axis $z = 0$ and the parabola (2.5) into the region between this parabola and the parabola

$$z = z_2 = \frac{2\gamma}{\gamma - 1}\left(V - \frac{2}{2 + \nu}\right)^2 \tag{3.2}$$

it is enough to show that the above mentioned integral curves (the ones that have to be transited to by a jump) lie entirely outside the above mentioned region.

The region containing these integral curves is limited on the left by the integral curve of the problem about a strong discontinuity (3.1) which has only one point common to the parabola (3.2) different from point

$V = 2/(2+\nu)$, $z = 0$:

$$V = 4/(\gamma+1)(2+\nu), \qquad z = 8\gamma(\gamma-1)/(\gamma+1)^2(2+\nu)^2$$

which proves our statement.

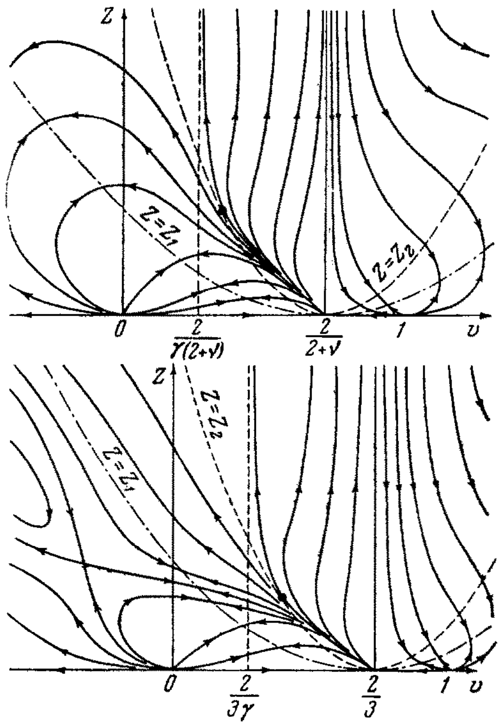

Fig. 1.

Case 2, $\nu = 1$. For this case the field of integral curves for conditions $1 < \gamma < 2$ is represented in Fig. 2. This case differs from the case when $\nu \neq 1$ by the presence of curves emanating from points $V = z = 0$ and $V = 2/3$, $z = 0$ and extending into infinity at $V \to \infty$. For this case the following asymptotic formulae hold

$$z \approx CV^2, \qquad \lambda \approx C_1 V^{\frac{3}{2}} \qquad (C,\ C_1 = \text{const}) \tag{3.3}$$

But limiting ourselves to the case of symmetrical initial distribution, we shall note that it is impossible to set up a solution continuable for all instants $t > 0$, because according to the indicated curves the mapped points cannot extend to infinity as according to (3.3) and (1.8) we would have $u(0,t) \neq 0$, which contradicts the obvious conditions of symmetry.

Now we arrive at the conclusion that the automodel problem of Cauchy for the case when the field of integral curves coincides with the field of the problem of a strong explosion, has no solution continuable for instants $t > 0$.

4. Let us consider the problem of automodel motion of gas that is being forced out by a symmetric piston expanding by a power law [3].

The field of integral curves for the problem about the piston coincides with the one about the problem of a strong explosion if $n = -\nu/(2+\nu)$. In order to set up the solution of the piston problem, we have to find the integral curve (either continuous or discontinuous) that goes through the point $V = z = 0$ (which corresponds to a region in the physical space, not yet disturbed) and some point of a straight line $V = 2/(2+\nu)$ (corresponding to the piston) [2.3] such that the parameter λ along it will vary monotonically. However, referring to Figs. 1 and 2 we can see that such curves do not exist. As established above, the region, which can be entered by a jump from the integral curves emanating from the origin of the coordinate system, touches the integral curve of the problem about a strong explosion in one point only, which is an image of the origin of coordinates.

Therefore the integral curves through which we can arrive at the straight line $V = 2/(2+\nu)$ remain outside of this region. Hence, the piston problem for $n = n_0 = -\nu/(2+\nu)$ does not have a solution. Let us see how the solution possibilities of the piston problem will be affected if n varies in the neighborhood of $n = n_0$. The study of the integral curve field for the piston problem reveals that with the value of n close to n_0 the character of singular points and the distribution of integral curves remain the same as with $n = n_0$. At $n = n_0$ curves (3.1) and (3.2) touch one another in the point

$$V = V_0 = \frac{4}{(\gamma+1)(2+\nu)}, \qquad z = z_0 = \frac{8\gamma(\gamma-1)}{(\gamma+1)^2(2+\nu)^2}$$

When the value of n is close to n_0, there exists an integral curve (only one) that goes through points $V = n+1$, $z = 0$ and $V = -2n/\nu\gamma$, $z = \infty$, this curve at $n = n_0$ coincides with curve (3.1). Just as for $n = n_0$, this curve bounds on its left a region of integral curves that emanate from the point $V = n+1$, $z = 0$ and go to point $V = n+1$, $z = \infty$. The parabola

$$z = \frac{2\gamma}{\gamma-1}[V-(n+1)]^2 \tag{4.1}$$

coincides with (3.2) at $n = n_0$, and bounds a region on its right into which a transition can be made by a jump from the integral curves that emanate from the origin of the coordinates.

At $n = n_0$ these curves have a common point of tangency and as the right-hand sides of these equations are analytical functions of parameter n, when n is close to n_0 , [for the first curve this follows from the analytical character of its [5] differential equation (1.4), for the second curve it is obvious from its implicit equation (4.1)], then the difference of these right-hand sides has an isolated zero for $V = \nu_0$ at the point $n = n_0$.

If it were possible to prove that this zero is simple, this would mean that if n varied in one direction from n_0 (namely increase) then as shown in calculations performed in Ref. [3] the curves intersect, that is in the above mentioned regions a common part is created and it becomes possible to set up a unique, discontinuous type of solution; with decrease in n, the curves no longer have any common points and the regions no longer have a common part. Therefore the solution ceases to exist.

In Ref. [3] for several values of $n > n_0$ a solution is set up, but with two values $n \leqslant n_0$ ($n = -0.5$, $\nu = 1$, $\nu = 2$) a solution was impossible to set up. In this work, a solution is set up (by means of numerical integration) for $n = -0.5$, $\nu = 3$, that is for $n - n_0 = 0.1$. This shows that if the above mentioned zero is not simple and not the first one encountered with decrease in n, then the next zero is larger than n_0 and is quite close to the first one (not further than 0.1).

In the article by Cherny [6] it was established that, during the solution of this problem by his approximate method, when n approaches n_0 from above, the terms of the series, that give the solution of the problem, become of the same order, that is the series becomes divergent. This fact does not imply that the approximate method with $n = n_0$ becomes inadequate for the solution of the problem, but that the solution for $n \leqslant n_0$ does not exist. On the other hand, as the divergence begins at exactly $n = n_0$ and not earlier (when n approaches n_0 from above), this shows (although it does not prove) that the zero under consideration is not a simple one and the first one from above, that is the solution of the problem ceases to exist exactly at $n = n_0$ and not earlier.

Based on the discussion above, we can formulate the following statement:

A solution of the problem of motion of an ideal, nonconducting perfect gas, displaced by a symmetrical piston that has a velocity $U = Ct^n$ for $-1 \leqslant n < -\nu/(2+\nu)$ does not exist.

At $n > -\nu/(2+\nu)$ one discontinuous solution of the problem exists.

5. The preceding considerations show that in working with some initial and initial-boundary problems about motions of an ideal, nonconducting perfect gas, it is impossible to set up a solution.

In these cases for initial problems the solution exists only near the $t = 0$ axis. In the space of variables of r and t limiting lines appear. We can call them "limiting" because of analogy to a similar phenomenon that is known for adiabatic steady state planar motions of gas. Here the solution cannot be continued beyond these lines. In the considered class of initial problems, the initial functions are unlimited at $r = \infty$ or at $r = 0$. Sedov stated that in the case when the initial functions are unlimited at $r = 0$, Cauchy's problem does not have a solution continuable for all instants of time $t > 0$. This is connected with the fact that either the initial mass or the initial energy of gas at $r = 0$ is unlimited. Investigations of special cases confirm this statement, which will be proven below. It would be interesting to check this for a general case.

It is easy to establish that the conditions for having finite initial mass and energy in the neighborhood of point $r = 0$ are $\alpha > -\nu$; $\beta > -\nu$. Let us consider Cauchy's problem with a field of integral curves coinciding with the field of problems about a piston for n close to n_0. Previously we established that the solution of this problem does not exist for $n = n_0$. From the discussion of the character of the field of integral curves near $n = n_0$ it follows that the solution of Cauchy's problem does not exist for $n < n_0$ but with $n > n_0$ a jump can be made onto integral curves going to point $V = n + 1$, $z = \infty$ and onto a single curve going to point $V = -2n/\nu\gamma$, $z = \infty$. Also if the two curves, between which the jump is made, are defined, the jump can be made in only one manner. Consideration of asymptotic formulae for solutions near point $V = n + 1$, $z = \infty$, shows that the solution with a jump on curves leading to that point, do not correspond to Cauchy's problem, but Cauchy's problem with a piston moving from the center of symmetry according to the law $\lambda(r, t) = \lambda^*$ as at point $V = n + 1$, $z = \infty$ gives finite values for λ and pressure.

To Cauchy's problem corresponds the jump onto the unique curve, as according to asymptotical formulae close to point $V = -2n/\nu\gamma$, $z = \infty$ we get that here $\lambda = \infty$, that is $r = 0$ and $u(0, t) = 0$. This shows us that at $n > n_0$ Cauchy's problem has a unique discontinuous solution and does not have any solution at $n \leqslant n_0$. In this case $n = n_0$ corresponds to $\alpha = -\nu$ that is the solution ceases to exist, when we violate the condition of limitation of initial energy in the neighborhood of $r = 0$, that is, the statement of Sedov is confirmed.

Let us investigate the solution for $n \to n_0 + 0$. It is easy to show that here the points on the integral curves from which the jump is made reduce to point $V = z = 0$. λ varies continuously on the jump. At point $V = z = 0$, $\lambda = 0$, this means that in the image of this point λ is also equal to zero. But this image is not a singular point, hence $\lambda = 0$ on the whole integral curve, onto which the jump was made for $n = n_0$, except for singular point $V = -2n_0/\nu\gamma$, $z = \infty$. Using the conditions for shock waves, we can find the distribution $z(V)$, $R(V)$, and $P(V)$ for $n = n_0$ (they are elementary functions and are given in [2]). For the transfer from argument V to λ we can use the function $V(\lambda)$.

For $n = n_0$ it is the following:

$V = -2n_0/\nu\gamma$, $\lambda \neq 0$ and $V = V$, $\lambda = 0$, which means that in all space, the quantities Z, P and R do not depend on λ. In order to find these quantities we must substitute $V = -2n_0/\nu\gamma$ into $Z(V)$, $R(V)$ and $P(V)$ (see formulas (7.14), (7.15), (7.16) and (7.17) Chapter 4 of Ref. [2]). This gives $Z = \infty$, $R = 0$, $P = \infty$. This is the limit approached by our solution for $n \to n_0 + 0$.

Similarly we can watch the behavior of the solution of the problem about a piston for $n \to n_0 + 0$. The limiting integral curve is then represented by (3.1) and the segment $z = \infty$, $-2n_0/\nu\gamma \leqslant V \leqslant n_0 + 1$. The distributions of λ and R along this segment are:

$$\lambda = \lambda_{n_0} = (n_0 + 1)\,(n_0 + 1 + 2n_0/\nu\gamma)^{-\frac{1}{\nu}}(V + 2n_0/\nu\gamma)^{\frac{1}{\nu}} \qquad (5.1)$$

$$R = c\,[(n_0 + 1 - V)/(V + 2n_0/\nu\gamma)]^{\frac{1}{\gamma-1}} \qquad (C = \text{const}) \qquad (5.2)$$

Here the condition λ for piston $= n_0 + 1$ is satisfied. From (5.1) we can see that $\lambda = 0$ for $V = -2n_0/\nu\gamma$, $z = \infty$. Since λ must increase monotonically along the whole integral curve which describes the solution, then $\lambda = 0$ on every part of the integral curve which coincides with (3.1).

The distribution of $Z(V)$, $R(V)$ and $P(V)$ along (3.1) should be mapped continuously on the distribution along segment $z = \infty$, $-2n_0/\nu\gamma \leqslant V \leqslant n_0 + 1$. This gives for constant C a value of zero because from $R(V)$ along (3.1) it follows $R(-2n_0/\nu\gamma) = 0$. The limiting solution will be $Z = \infty$, $P = \infty$, $\lambda = \lambda_{n_0}$.

The meaning of that solution is as follows: because on the shock wave $\lambda = 0$, the shock wave is going to infinity instantaneously, leaving distributions of parameters in the form

$$u = \frac{r}{t} V(\lambda)|_{t \to +0} = \infty, \qquad p = \infty, \qquad \rho = 0$$

Because here, as everywhere, the pressure on the piston is equal to infinity, then to accomplish its motion according to the prescribed law with $n = n_0$ it is necessary to apply an infinite amount of work. Thus, to move piston for $n \leqslant n_0$ according to $r = [c/(n+1)]\; t^{n+1}$ is impossible i.e. making the statement of a problem for these values of n loses all meaning, which shows the senselessness of solving the piston problem for $n < n_0$.

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